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HEAT TRANSFER BY THERMAL CONVECTION AT HIGH RAYLEIGH NUMBERS

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NOMENCLATURE

$C_1, C_2, C_3, C_4, C_5, C_6$, constants in correlation equations;
m, n, exponents in correlation equations;
N, number of measured values;
Nu, Nusselt number;
Nu_{meas}, measured value of Nusselt number;
Nu_{calc}, Nusselt number calculated from correlation;
r^2, coefficient of determination for correlation;
Pr, Prandtl number;
Ra, Rayleigh number.

Greek symbol

δ, rms deviation:

$$\delta = \left[\frac{1}{N}\sum_{1}^{N}\left(\frac{Nu_{\text{meas}} - Nu_{\text{calc}}}{Nu_{\text{calc}}}\right)^2\right]^{1/2} \cdot 100\%.$$

INTRODUCTION

SEVERAL investigators [1, 2] recently presented theoretical expressions for the variation of Nusselt number with Rayleigh number for turbulent thermal convection across a horizontal layer heated from below. Unfortunately, experimental data to confirm analytical predictions at high Rayleigh numbers are relatively sparse.

Threlfall [3] measured the heat transfer across a layer of gaseous helium at about 3 K for Rayleigh numbers up to 2×10^9. Fitzjarrald [4], using air in a relatively large apparatus, took measurements up to a Rayleigh number of 7.4×10^9. In experiments using a layer of water [5], the heat transfer was measured at Rayleigh numbers up to 3.3×10^9. The present results are an extension of the study reported in [5].

FORM OF CORRELATION EQUATIONS

Many empirical and analytical predictions of the Nusselt-number variation at high Rayleigh number are of the form

$$Nu = C_1 Ra^n. \tag{1}$$

In some analyses, the exponent on the Rayleigh number, n, is one-third:

$$Nu = C_2 Ra^{1/3}. \tag{2}$$

Equation 2 predicts that the heat transfer is independent of the thickness of the layer; the resistance to heat transfer essentially occurs only near the upper and lower boundaries.

Related expressions which attempt to take into account the fact that the Nusselt number is unity at low Rayleigh numbers are

$$(Nu - 1) = C_3 Ra^m \tag{3}$$

and

$$(Nu - 1) = C_4 Ra^{1/3}. \tag{4}$$

Sometimes the value of the Rayleigh number is reduced by the critical value in such correlations, but at high Rayleigh numbers, this would have little effect on the prediction.

A somewhat more comlex relationship presented by Long [2] also predicts a one-third power law variation at very high Rayleigh number but might fit the data better at lower Rayleigh numbers. Hollands and Raithby [6] present an empirical relationship to fit heat-transfer data over a range of conditions from pure conduction to high Rayleigh number turbulent flow.

APPARATUS AND EXPERIMENTS

An apparatus in the form of a vertical circular cylinder is used in the present study. A 3 mm thick, 450 mm I.D. Plexiglass tube, insulated on the outside, forms the side boundary of the fluid layer. The lower boundary is a flat 12.5 mm thick copper plate with electrical heating elements placed beneath it. Guard heaters surround the bottom plate to ensure proper knowledge of the heat flow into the working fluid. The upper boundary is a 25 mm thick copper plate with cooling water channels cut into the upper surface to remove the heat transmitted through the fluid layer. Both copper plates were nickel-plated and then chrome-plated.

The temperature difference across the fluid layer is determined from readings of multi-junction thermopiles. The power input to the fluid is determined by the electrical power input to the heater on the bottom plate. Demineralized water serves as the test fluid; its properties are determined at the average temperature in the fluid layer. Precision spacers control the distance between the upper and lower plates and therefore the thickness of the water layer. In the present tests, the spacing is varied between 100 and 787 mm.

RESULTS

The data obtained in the tests are shown in Table 1 and Fig. 1. The Rayleigh number range extends from about 10^8 up to 2.26×10^{11}. The corresponding Nusselt number varies from 27 to 358. Figure 1 also includes data from [5] as well as the correlation in the form of equation (2) for the high Rayleigh-number data.

Table 1. Heat transfer measurements

L(mm)	ΔT(°C)	$Ra \times 10^{-8}$	Nu	Pr
100	4.40	0.999	27.2	5.71
180	2.88	2.64	36.1	6.76
180	4.99	5.00	43.8	6.50
180	5.66	6.64	48.6	6.05
180	6.62	8.04	51.3	5.95
180	7.75	9.49	53.6	5.74
180	2.46	2.33	34.3	6.65
180	2.84	2.75	33.4	6.60
180	2.74	2.63	37.8	6.61
180	3.01	2.85	37.8	6.66
180	3.16	3.01	39.0	6.63
180	3.36	3.23	39.5	6.61
180	3.48	3.35	40.4	6.60
180	3.82	3.62	39.5	6.65
180	4.01	3.83	40.0	6.63
180	4.21	4.05	40.4	6.60
180	4.31	4.16	41.6	6.59
180	4.64	4.40	40.5	6.65
180	4.70	4.48	42.1	6.63
180	4.86	4.66	42.6	6.61
180	4.94	4.76	43.9	6.60
180	5.16	4.88	43.8	6.65
180	5.32	5.07	44.5	6.63
180	5.44	5.21	45.1	6.62
180	5.66	5.46	45.1	6.60
180	5.75	5.58	45.9	6.58
180	6.02	5.71	45.6	6.64
180	6.12	5.84	46.2	6.63
180	6.30	6.04	46.6	6.61
180	6.47	6.14	46.7	6.64
180	6.67	6.35	46.9	6.63
180	6.72	6.44	48.0	6.61
180	6.86	6.53	48.2	6.64
180	7.14	6.76	47.9	6.64
180	7.34	7.01	48.7	6.62
180	7.48	7.20	49.2	6.60
180	7.66	7.25	49.4	6.65
180	7.78	7.43	50.1	6.62
180	7.99	7.68	50.4	6.61
180	8.12	7.86	50.9	6.59
180	8.22	7.98	50.9	6.60
180	8.28	8.05	52.0	6.57
180	8.38	7.97	52.6	6.64
180	8.55	8.19	52.6	6.61
180	8.68	8.23	52.6	6.64
180	8.81	8.33	52.9	6.65
180	8.95	8.53	53.4	6.63
180	9.93	9.27	49.4	6.69
180	9.48	9.05	53.0	6.63
180	9.51	9.10	54.2	6.62
180	9.52	9.16	55.1	6.60
180	9.77	9.27	55.2	6.64
180	9.74	9.36	56.7	6.61
460	3.07	51.5	94.1	6.48
460	4.12	69.4	106.0	6.47
460	6.16	122	126.0	6.03
460	9.01	187	146.0	6.00
670	3.88	205	150.0	6.43
670	5.67	307	174.0	6.36
670	7.34	388	188.0	6.43
670	9.06	480	202.0	6.43
670	16.33	785	238.0	6.32
787	7.32	614	219.0	6.45
787	11.15	957	260.0	6.43
*787	13.60	1180	293.0	6.44
*787	19.87	1640	324.0	6.54
787	23.99	2260	358.0	6.17

*Suspect points (see text).

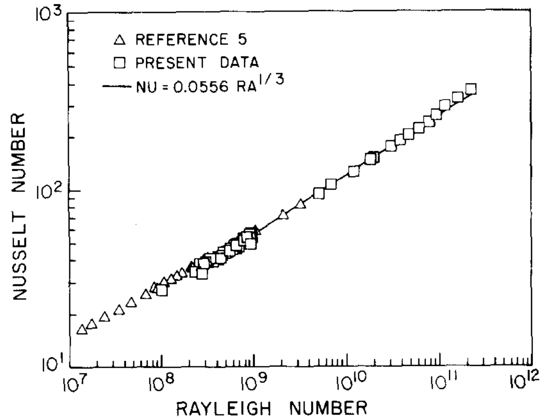

FIG. 1. Nusselt number as a function of Rayleigh number.

Note that during the tests, the results from two of the relatively high Rayleigh-number runs were considered suspect because of temporal variations in the readings. To allow time for fluid motions to settle out, the system is usually kept at room temperature for 2–3 days (sometimes for seven) before starting a test at the larger spacings. Once tests begin, the system is maintained at the desired temperature difference for 2 or 3 days before final measurements are taken. The two exceptions to this procedure (at large spacing) are the runs at Rayleigh numbers of 1.18×10^{11} and 1.64×10^{11}. For these tests, the preliminary period was only about 3 h.

A least-squares analysis of the present data and data from [5] is used to obtain the constants in equations (1) and (2). Various portions of the Rayleigh number range yield somewhat different values as shown in Table 2. In some of the correlations, the data for the two suspect runs are excluded; this results in only slight changes in the empirical constants.

The correlation of [6] is quite serviceable over a large range of Rayleigh numbers but does not fit the data quite as well as do the simple forms of equations (1)–(4). At Rayleigh number less than 2×10^9 the data lie about 5% below the correlation of [3].

Fitting the data to equation (1) yields an exponent between 0.334 and 0.339. It appears reasonable to force the correlation to fit a one-third power law. This has only a slight effect on the coefficient of correlation as compared to the best fit with equation (1). Using the present data at high Rayleigh number,

$$Nu = 0.0556\, Ra^{1/3}. \quad (5)$$

The rms deviation of the data above Rayleigh numbers of 10^9 from this equation is about 2%.

It should be borne in mind that in the present tests, a layer thickness as large as 790 mm is used, while the diameter of the layer is 450 mm. At this aspect ratio, the side boundaries probably influence the flow pattern. Previous work [7] suggests that this might not greatly affect the mean heat-transfer rate. Further examination of the influence of the aspect ratio will be made in future studies.

In summary, measurements of the heat transfer across a horizontal layer of water at high Rayleigh number appear to fit a prediction that the Nusselt number varies as the Rayleigh number to the one-third power.

Table 2.

Data	No. of data points	C_1	m	r^2	δ (%)
A: Data correlated by $Nu = C_1 Ra^n$ [equation (1)]					
All of present data	67	0.0527	0.336	0.9981	2.7
Present data less two suspect points	65	0.0548	0.335	0.9980	2.5
Present data for $Ra > 5 \times 10^8$	49	0.0492	0.339	0.9984	2.2
Present data for $Ra > 5 \times 10^8$ less two suspect points	47	0.0524	0.336	0.9988	1.9
Data of [5] plus present data less two suspect points for $Ra > 10^9$	15	0.0487	0.339	0.9988	1.7
B: Data correlated by $Nu = C_2 Ra^{1/3}$ [Equation (2)]					
		C_2	r^2	δ (%)	
All of present data	67	0.0557	0.9980	2.7	
Present data less two suspect points	65	0.0556	0.9980	2.5	
Present data for $Ra > 5 \times 10^8$	49	0.0556	0.9986	2.4	
Present data for $Ra > 5 \times 10^8$ less two suspect points	47	0.0555	0.9988	2.0	
Data of [5] plus present data less two suspect points for $Ra > 10^9$	15	0.0556	0.9986	1.9	

Suspect points of present data are at Rayleigh numbers of 1.18×10^{11} and 1.64×10^{11}; δ is the rms deviation. r^2 is coefficient of determination for best fit of correlation.

REFERENCES

1. J. Toomre, D. O. Gough and E. H. Spiegel, Numerical solution of single-mode equations, *J. Fluid Mech.* **79**, 1 (1977).
2. R. R. Long, Relation between Nusselt number and Rayleigh number in turbulent thermal convection, *J. Fluid Mech.* **73**, 445 (1976).
3. D. C. Threlfall, Free convection in low-temperature gasoline helium, *J. Fluid Mech.* **67**, 17 (1975).
4. D. E. Fitzjarrald, An experimental study of turbulent convection in air, *J. Fluid Mech.* **73**, 693 (1976).
5. A. M. Garon and R. J. Goldstein, Velocity and heat transfer measurements in thermal convection, *Physics Fluids* **16**, 1818 (1973).
6. K. G. T. Hollands, G. D. Raithby and L. Konicek, Correlation equations for free convection heat transfer in horizontal layers of air and water, *Int. J. Heat Mass Transfer* **18**, 879 (1975).
7. I. Catton and D. K. Edwards, Effect of side walls on natural convection between horizontal plates heated from below, *J. Heat Transfer* **89**, 295 (1967).

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HEAT TRANSFER CHARACTERISTICS FOR LAMINAR FLOW BETWEEN PARALLEL PLATES WITH SUCTION

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NOMENCLATURE

D, half-width of the two-dimensional channel;
l, nondimensional length of the channel, L/D;
L, length of the channel;
Pe, Peclet number, $U_{be}D/\alpha$;
Re_e, inlet Reynolds number, $U_{be}D/\nu$;
Re_w, suction Reynolds number, $V_w/D/\nu$;
T, temperature;
u, nondimensional horizontal velocity, U/U_{be};
U, horizontal velocity;
v, nondimensional vertical velocity, V/U_{be};
V, vertical velocity;
V_w, suction velocity, equal to $-V$ at the wall;
x, nondimensional horizontal length coordinate, X/D;